Harmonic analysis of irradiation asymmetry for cylindrical implosions driven by rotating ion beams

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Abstract

Cylindrical implosions driven by intense heavy ions beams should be instrumental in a near future to study High Energy Density Matter. By rotating the beam by means of a high frequency wobbler, it should be possible to deposit energy in the outer layers of a cylinder, compressing the material deposited in its core. The beam temporal profile should however generate an inevitable irradiation asymmetry likely to feed the Rayleigh-Taylor instability (RTI) during the implosion phase. In this paper, we compute the Fourier components of the target irradiation in order to make the junction with previous works on RTI performed in this setting. Implementing a 1D and 2D beam models, we find these components can be expressed exactly in terms of the Fourier transform of the temporal beam profile. If T is the beam duration and Ω its rotation frequency, "magic products" ΩT can be identified which cancel the first harmonic of the deposited density, resulting in an improved irradiation symmetry.

Key words: Intense particle beams and radiation sources, Charged-particle beams, High-pressure effects in solids and liquids
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1 Introduction

The study of matter under extreme conditions of pressure and density is of great interest for astrophysics, planetary sciences and inertial fusion [1–4]. Particularly appealing is the perspective to produce cylindrical implosions with a high degree of symmetry. Such experiments would allow to generate large volumes of high-energy density matter (HEDM), including strongly coupled plasmas. Among others, the experimental realization of the once predicted metallic state of Hydrogen [5] is envisioned at the Gesellschaft für Schwerionenforschung (GSI) near Darmstadt, Germany. Within the framework of the Facility for Antiproton and Ion Research (FAIR) currently under construction at GSI [6], the so-called LAPLAS experiment (LAboratory of PLAnetary Sciences) aims at implementing such cylindrical scheme to study equation of state and transport properties of HEDM [7–9].

Figure 1 sketches the typical experimental scheme implemented for cylindrical implosions. A hollow ion beam hits the absorber part of the cylinder. By tailoring its energy so that the Bragg peak of the ions falls well outside the cylinder, it is possible to obtain a very homogenous energy deposition inside the cylinder. The following compression has been the object of various numerical and analytic studies in recent years, to assess, among other, the impact of the Rayleigh-Taylor instability (RTI) on the medium interfaces during the compression [10–14,4].

The hollow beam generation should be achieved by a high frequency wobbler rotating the ion beam [15]. The rotation frequency necessary to achieve an acceptable degree of irradiation symmetry has been calculated by Piriz *et al.* [12] and found in the GHz range. For such a fast deposition, the target motion during the deposition can be neglected. But even so, an inevitable source of asymmetry comes from the beam temporal profile itself. Because the beam duration is finite, the number of ions deposited along the absorber will be inhomogeneous. The overall effect of such asymmetry on the pressure driving the compression has been investigated in Refs. [11] and [13], where the amplitude of the asymmetry has been evaluated. But previous works on the RTI for this setting have showed that stability is a matter of both the excited wavelength, and the excitation amplitude. It is thus necessary to know the Fourier components of the ion deposition in the pusher, precisely because RTI analysis is performed in Fourier space.

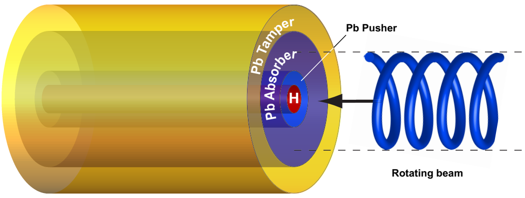

Fig. 1. (Color online) Typical cylindrical implosion scheme. Not to scale.

2 1D beam model

Consider a 1D ion beam with temporal profile $I(t)$, normalized to unity so that $\int I(t)dt = 1$. The beam is rotated by a wobbler over the absorber region at velocity Ω rad/s (see Figure 2). The number of ions brought by the beam between t and $t + dt$ is $I(t)dt$. Suppose ions arriving at $t = 0$ hit the target at $\theta = 0$. Then those arriving at $t = \theta/\Omega$ hit the target at angle θ, as explained on the Figure. The number of ions deposited between θ and $\theta + d\theta$, from time t to time $t + dt$ is $I(\theta/\Omega)d\theta/\Omega$.

Then, ions arriving $2l\pi/\Omega$ seconds later, or before, hit the very same point, for any l integer, positive or negative. The total number of ions eventually deposited between θ and $\theta + d\theta$ is thus

$$dN(\theta) = \sum_{l=-\infty}^{\infty} I\left(\frac{\theta}{\Omega} + \frac{2l\pi}{\Omega}\right)\frac{d\theta}{\Omega}. \quad (1)$$

As expected, the density deposition $dN(\theta)/d\theta \equiv \rho(\theta)$ is periodic, of period 2π.

We now turn to the Fourier transform of the density deposition $\rho(\theta)$ in the absorber ring,

$$\widehat{\rho}(s) = \frac{1}{\Omega}\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty} I\left(\frac{\theta + 2l\pi}{\Omega}\right)e^{is\theta}d\theta. \quad (2)$$

Permuting the infinite sum and making the substitution $u = \theta/\Omega + 2l\pi/\Omega$, we find

$$\widehat{\rho}(s) = \sum_{l=-\infty}^{\infty}\left(\int_{-\infty}^{\infty} I(u)e^{is\Omega u}du\right)e^{-2il\pi s}. \quad (3)$$

The term between parenthesis is just the Fourier transform of the beam tem-

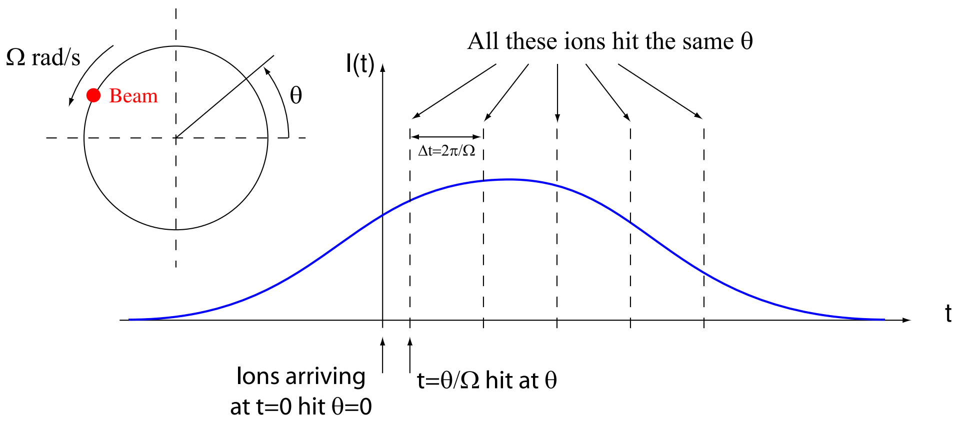

Fig. 2. (Color online) The beam intensity profile is $I(t)$. It rotates at velocity Ω rad/s along the circle. Ions arriving at $t = \theta/\Omega + 2k\pi/\Omega$ all hit the same point of the circle.

poral profile for the frequency value "$s\Omega$". We thus have

$$\widehat{\rho}(s) = \sum_{l=-\infty}^{\infty} \widehat{I}(s\Omega)e^{-2il\pi s} = \widehat{I}(s\Omega) \sum_{l=-\infty}^{\infty} e^{-2il\pi s}. \tag{4}$$

We now use $\sum_{l=-\infty}^{\infty} e^{-2il\pi s} = \sum_{l=-\infty}^{\infty} \delta(s-l)$ [16] and find,

$$\widehat{\rho}(s) = \sum_{l=-\infty}^{\infty} \widehat{I}(s\Omega)e^{-2il\pi s} = \widehat{I}(s\Omega) \sum_{l=-\infty}^{\infty} \delta(s-l). \tag{5}$$

The infinite sum $\sum \delta(s-l)$ functions at the right hand side implies the spectrum is discrete. This comes from the fact that we consider the Fourier transform of the periodic function $\rho(\theta)$. Furthermore, the sum of δ's implies that the discrete values of the spectrum are the integers $l = -\infty \ldots \infty$. The amplitudes of the harmonics are therefore,

$$\begin{aligned} \widehat{\rho}(0) &= \widehat{I}(0), \\ \widehat{\rho}(l) &= \widehat{I}(l\Omega) + \widehat{I}(-l\Omega), \quad \forall l \in \mathbb{N}^*. \end{aligned} \tag{6}$$

Such is the final result, where the amplitude of the harmonics of the density deposition is expressed in terms of the beam temporal profile. A 2D model accounting for a transverse extension of a beam of radius R_b can be developped [17] found that the 1D approximation holds while $R_b \ll R_c$, where R_c is the radius of the circle described by the center of the beam.

Noteworthily, the stability criterion of [14] can be retrieved computing the amplitude of the first harmonic, and stating that this mode alone has to be stable.

3 Canceling the first harmonic

An important issue for the present problem is the presence of the first harmonic. One reason is that its amplitude is usually the largest one. But most of all, the relevant RTI analysis has been developed for a planar interface. If the interface is circular, the planar analysis is still valid providing the wavelength of the perturbation is much smaller than the circumference of the circle. In the present case, the first excited wavelength λ_1 is precisely the circumference. It is thus probable that the planar analysis fails for this mode. Canceling the first harmonic could solve the two problems at once: the planar analysis would just be applied to the next modes $\lambda_1/2$, $\lambda_1/4\ldots$, with a better accuracy and also, we could reduce significantly the irradiation asymmetry.

Canceling the first harmonic means tailoring the pulse shape $I(t)$ and the rotation frequency Ω to fulfill,

$$\int_{-\infty}^{\infty} I(t)\cos(\Omega t)dt = 0. \tag{7}$$

This quadrature is an oscillating integral vanishing for $\Omega \to \infty$. If it does so oscillating around 0, there will be an infinite number of values of Ω canceling the first harmonic.

Consider for example the parabolic time profile

$$\begin{aligned} I(t) &= \frac{3}{2T}\left[1-\left(\frac{t}{T/2}\right)^2\right], \text{ for } -T/2 < t < T/2 \\ &= 0 \text{ otherwise.} \end{aligned} \tag{8}$$

The first harmonic amplitude vanishes if,

$$\frac{\Omega T}{2} = \tan\frac{\Omega T}{2}, \tag{9}$$

which has an infinite number of solutions. The first ones can be found numerically. Then, the solutions are found just before $x_k = (2k+1)\pi/2$, with $k \in \mathbb{N}$. Considering $\tan x \sim (x_k - x)^{-1}$ near x_k, we can find an approximation for x_k. The "magic values" of the product $\Omega T/2$ canceling the first harmonic are eventually given by,

$$\begin{aligned} x_1 &= 4.493, \\ x_2 &= 7.725, \end{aligned}$$

$$
\begin{aligned}
x_3 &= 10.904, \\
&\dots \\
x_k &= (2k+1)\frac{\pi}{2} - \frac{1}{(2k+1)\pi/2} + \mathcal{O}(1/k^2).
\end{aligned} \tag{10}
$$

By choosing any of these $\Omega T = 2x_k$, the main asymmetry will come from the second harmonic only. The present procedure can be adapted to any beam profile and should allow to determine the optimum stability setup accordingly.

4 Conclusion

We have conducted the Fourier analysis of the ion density deposited on the circular absorber by a rotating beam. For a point-like beam with temporal profile, calculation can be performed exactly, and the spectrum of the deposited ion density expressed in terms of the spectrum of the temporal profile.

It is possible to cancel the first harmonic of the beam deposition around the absorber. This allows to further reduce the irradiation asymmetry, and renders more realistic the use of a RTI planar interface analysis. Calculations have been performed selecting a parabolic temporal beam profile, but they should eventually be conducted accounting for the beam profile chosen for the experiment.

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